Topic:- DU_J19_MA_MATHS

1) The order of Sylow subgroups of a finite group G of order 56 are [Question ID = 24519]

1. 2 and 28 [Option ID = 38076]
2. 7 and 8 [Option ID = 38074]
3. 8 and 14 [Option ID = 38077]
4. 4 and 14 [Option ID = 38075]

Correct Answer :-

- 7 and 8 [Option ID = 38074]

2) The remainder when 5^{2019} is divided by 11 is [Question ID = 24520]

1. 6 [Option ID = 38080]
2. 9 [Option ID = 38081]
3. 1 [Option ID = 38078]
4. 4 [Option ID = 38079]

Correct Answer :-

- 1 [Option ID = 38078]

3) The smallest positive integer n, which leaves remainders 2,3 and 4 when divided by 5,7 and 11 respectively, is [Question ID = 24521]

1. 751 [Option ID = 38083]
2. 1136 [Option ID = 38085]
3. 176 [Option ID = 38082]
4. 367 [Option ID = 38084]

Correct Answer :-

- 176 [Option ID = 38082]

4) Suppose that the equation $x^2 \cdot a \cdot x = a^{-1}$ is solvable for a in a group G. Then, there exists b in G such that

[Question ID = 24515]

1. $a = b^3$ [Option ID = 38059]
2. $a = b^5$ [Option ID = 38061]
3. $a = b^4$ [Option ID = 38060]
4. $a = b^2$ [Option ID = 38058]

Correct Answer :-

- $a = b^2$ [Option ID = 38058]

5) Consider the following statements:

(i) Every metric space is totally bounded.
(ii) A totally bounded metric space is bounded.

Then

[Question ID = 24536]

1. neither (i) nor (ii) is true [Option ID = 38145]
2. only (ii) is true [Option ID = 38143]
3. only (i) is true [Option ID = 38142]
4. both (i) and (ii) are true [Option ID = 38144]

Correct Answer :-

- only (i) is true [Option ID = 38142]

6)

Consider the following statements:

(i) Every minimal generating set of a vector space is a basis.
(ii) Every maximal linearly independent subset of a vector space is a basis.
(iii) Every vector space admits a basis.

Then

[Question ID = 24510]

1. all of (i), (ii) and (iii) are true [Option ID = 38041]
2. only (i) and (ii) are true [Option ID = 38038]
3. only (i) and (iii) are true [Option ID = 38040]
4. only (ii) and (iii) are true [Option ID = 38039]

Correct Answer :-
- only (i) and (ii) are true [Option ID = 38038]

7) The differential equation of a family of parabolas with foci at origin and axis along x-axis is

[Question ID = 24506]

1. $y\left(\frac{dy}{dx}\right)^2 + 2x^2\frac{dy}{dx} + y = 0$ [Option ID = 38023]
2. $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$ [Option ID = 38024]
3. $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} + y = 0$ [Option ID = 38025]
4. $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y^2 = 0$ [Option ID = 38022]

Correct Answer :-
- $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y^2 = 0$ [Option ID = 38022]

8) Number of iterations required to solve $x^3 + 4x^2 - 10 = 0$ using bisection method with accuracy 10^{-3} (with initial bracket $[1, 2]$) are

[Question ID = 24495]

1. 7 [Option ID = 37978]
2. 12 [Option ID = 37981]
3. 10 [Option ID = 37980]
4. 8 [Option ID = 37979]

Correct Answer :-
- 7 [Option ID = 37978]

9) Let $P_2(t)$ denote the set of all polynomials over $\mathbb{R}$ of degree at most 2. With respect to the inner product

$$\langle p, q \rangle = \int_{-1}^{1} p(t)q(t)dt,$$

the set of vectors $\{1, t, t^2 - \frac{1}{3}\}$ is

[Question ID = 24513]

1. not a linearly independent set [Option ID = 38053]
2. orthogonal basis of $P_2(t)$ [Option ID = 38050]
3. basis of $P_2(t)$ but not orthogonal [Option ID = 38052]

4. orthogonal but not a basis of $P_2(t)$ [Option ID = 38051]

Correct Answer :-

- orthogonal basis of $P_2(t)$ [Option ID = 38050]

10) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be periodic if there exists $p > 0$ such that $f(x+p) = f(x)$, for all $x \in \mathbb{R}$. If f is a continuous periodic function on $\mathbb{R}$, then

[Question ID = 24543]

1. f^2 is unbounded [Option ID = 38173]
2. $|f|$ is unbounded [Option ID = 38170]
3. $|f|$ is not uniformly continuous [Option ID = 38172]
4. f^2 is uniformly continuous and bounded on $\mathbb{R}$ [Option ID = 38171]

Correct Answer :-

- $|f|$ is unbounded [Option ID = 38170]

11) Consider the following statements:

(i) Every separable metric space is compact.

(ii) Every compact metric space is separable.

Then

[Question ID = 24534]

1. only (i) is true [Option ID = 38134]
2. only (ii) is true [Option ID = 38135]
3. both (i) and (ii) are true [Option ID = 38136]
4. neither (i) nor (ii) is true [Option ID = 38137]

Correct Answer :-

- only (i) is true [Option ID = 38134]

12) The partial differential equation $x^3 u_{xx} - (y^2-1)u_{yy} = u_x$ is

[Question ID = 24502]

1. parabolic in $\{(x,y) \mid y < 0\}$ [Option ID = 38006]
2. elliptic in $\mathbb{R}^2$ [Option ID = 38008]
3. hyperbolic in $\{(x,y) \mid x > 0\}$ [Option ID = 38007]
4. parabolic in $\{(x,y) \mid y > 0\}$ [Option ID = 38009]

Correct Answer :-

- parabolic in $\{(x,y) \mid y < 0\}$ [Option ID = 38006]

13) Consider the following statements

(i) $\mathbb{Z}[x]$ is a principal ideal domain.

(ii) If R is a principal ideal domain, then every subring of R containing 1 is also a principal ideal domain.

Then

[Question ID = 24522]

1. only (i) is true [Option ID = 38086]
2. both (i) and (ii) are true [Option ID = 38088]
3. only (ii) is true [Option ID = 38087]
4. neither (i) nor (ii) is true [Option ID = 38089]

Correct Answer :-
- only (i) is true [Option ID = 38086]

14) Let $N \neq \{e\}$ be a normal subgroup of a non-abelian group G such that $N \cap G' = \{e\}$, where G' is the commutator subgroup of G. Then

[Question ID = 24517]

1. None of these [Option ID = 38069]
2. N is not abelian [Option ID = 38067]
3. $N \subseteq Z(G)$, the centre of G [Option ID = 38068]
4. G/N is abelian [Option ID = 38066]

Correct Answer :-
- G/N is abelian [Option ID = 38066]

15) Let $f(t) = t^2 e^t \log t;\ 1 \leq t \leq 3$. Then there exists some $c \in (1,3)$ such that $\int_1^3 f(t)dt$ is equal to

[Question ID = 24525]

1. $\frac{1}{3} e^c \log c^{26}$ [Option ID = 38098]
2. $c^2 e^c \log 3$ [Option ID = 38101]
3. $2^2 c^2 \log c$ [Option ID = 38099]
4. $26 e^c \log c$ [Option ID = 38100]

Correct Answer :-
- $\frac{1}{3} e^c \log c^{26}$ [Option ID = 38098]

16) For two ideals I and J of a commutative ring R define $(I : J) = \{r \in R \mid rI \subseteq J\}$. Then for the ring $\mathbb{Z}$ of integers what is $(8\mathbb{Z} : 12\mathbb{Z})$

[Question ID = 24523]

1. $4\mathbb{Z}$ [Option ID = 38093]
2. $\mathbb{Z}$ [Option ID = 38090]
3. $2\mathbb{Z}$ [Option ID = 38091]
4. $3\mathbb{Z}$ [Option ID = 38092]

Correct Answer :-
- $\mathbb{Z}$ [Option ID = 38090]

17) Consider the set $\mathbb{R}^2$ with metric defined by

$$d(x,y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}; \quad x = (x_1, x_2),\ y = (y_1, y_2).$$

Then which of the following set is not connected

[Question ID = 24535]

1. $\{(x,y) \in \mathbb{R}^2 \mid y^2 = x\}$ [Option ID = 38138]
2. $\{(x,y) \in \mathbb{R}^2 \mid x^2 - y^2 = 1\}$ [Option ID = 38141]
3. $\{(x,y) \in \mathbb{R}^2 \mid \frac{x^2}{4} + \frac{y^2}{9} = 1\}$ [Option ID = 38140]
4. $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ [Option ID = 38139]

Correct Answer :-

- $\{(x,y) \in \mathbb{R}^2 \mid y^2 = x\}$ [Option ID = 38138]

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18) Let $f(x) = \lim_{n\to\infty} \frac{n^x - n^{-x}}{n^x + n^{-x}}, x \in \mathbb{R}$. Then

[Question ID = 24542]

1. f is continuous at $(1, \infty)$ [Option ID = 38169]
2. f is not differentiable at $x = 1$ [Option ID = 38168]
3. f is not continuous at $x = -1$ [Option ID = 38167]
4. f is continuous at $x = 0$ [Option ID = 38166]

Correct Answer :-

- f is continuous at $x = 0$ [Option ID = 38166]

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19) For $x \in [-1, 1]$, let

$$f(x) = \begin{cases} x \operatorname{sgn}(\sin \frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0, \end{cases}$$

where sgn denotes the signum function. Then

[Question ID = 24526]

1. f is continuous on $[-1, 1]$ [Option ID = 38104]
2. f is not differentiable at any point of $[-1, 1]$ [Option ID = 38103]
3. f is Riemann integrable on $[-1, 1]$ [Option ID = 38102]
4. the set of points of discontinuity of f in $[-1, 1]$ is finite [Option ID = 38105]

Correct Answer :-

- f is Riemann integrable on $[-1, 1]$ [Option ID = 38102]

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20) The integral surface of the partial differential equation $p^2 + q^2 = 2$ which passess through $x = 0,\ z = y$ is

[Question ID = 24503]

1. $x^2 + y^2 + z^2 = 1$ [Option ID = 38013]
2. $z = y \pm x$ [Option ID = 38010]
3. $z^2 = x \pm y^2$ [Option ID = 38011]
4. $z^3 = x \pm y$ [Option ID = 38012]

Correct Answer :-

- $z = y \pm x$ [Option ID = 38010]

21) Does the sequence $a_n = n^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$ has a limit?

[Question ID = 24529]

1. No, it oscillates [Option ID = 38115]
2. No, it diverges [Option ID = 38114]
3. Yes, −2 is the limit [Option ID = 38117]
4. Yes, −1 is the limit [Option ID = 38116]

Correct Answer :-

- No, it diverges [Option ID = 38114]

22) The orthogonal trajectory of the family of curves $ay^2 = x^3$, where a is an arbitrary constant, is

[Question ID = 26021]

1. $3y^2 + 2x^2 = \text{constant}$ [Option ID = 44082]
2. $2y^2 - 3x^2 = \text{constant}$ [Option ID = 44080]
3. $3y^2 - 2x^2 = \text{constant}$ [Option ID = 44079]
4. $2y^2 + 3x^2 = \text{constant}$ [Option ID = 44081]

Correct Answer :-

- $3y^2 - 2x^2 = \text{constant}$ [Option ID = 44079]

23) The integral surface of the linear partial differential equation

$$xp + yq = z$$

which contains the circle defined by $x^2 + y^2 + z^2 = 4$, $x + y + z = 2$, is

[Question ID = 24504]

1. $\frac{x}{y} + \frac{z}{x} + \frac{y}{z} + 1 = 0$ [Option ID = 38015]
2. $xy + xz + yz = 0$ [Option ID = 38016]
3. $xy^2 + xz^2 = 0$ [Option ID = 38014]
4. $xyz = 1$ [Option ID = 38017]

Correct Answer :-

- $xy^2 + xz^2 = 0$ [Option ID = 38014]

24) Initial estimate for the root of the equation $f(x) = 0$ is $x_0 = 2$ and $f(2) = 4$. The tangent line to $f(x)$ at $x_0 = 2$ makes an angle of 42^0 with the x axis. The next estimate of the root by Newton-Raphson method is approximately

[Question ID = 24499]

1. 2.0102 [Option ID = 37995]
2. 4.4424 [Option ID = 37997]
3. 0.2412 [Option ID = 37994]
4. −2.4424 [Option ID = 37996]

Correct Answer :-

- 0.2412 [Option ID = 37994]

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25) The numerical scheme using the first three terms of the Taylor series for solving the differential equation

$$\frac{dy}{dx} + y = e^{-3x}, \quad y(0) = 5,$$

with $h = x_{i+1} - x_i$, is given by

[Question ID = 24497]

1. $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2}(-3e^{-3x_i} - y_i)$ [Option ID = 37988]
2. $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2}(-4e^{-3x_i} + y_i)$ [Option ID = 37987]
3. $y_{i+1} = y_i - h(e^{-3x_i} - y_i) + \frac{h^2}{2}(y_i - e^{-3x_i})$ [Option ID = 37989]
4. $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2} y_i$ [Option ID = 37986]

Correct Answer :-

- $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2} y_i$ [Option ID = 37986]

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26) Let $X = \mathbb{C}^n$, $0 < p < 1$ and $q = 1/p$. For $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in X define

$$d_p(x, y) = \Big(\sum_{i=1}^{n} |x_i - y_i|^p \Big)^{1/p}$$

and

$$d_q(x, y) = \Big(\sum_{i=1}^{n} |x_i - y_i|^q \Big)^{1/q}.$$

Then

[Question ID = 24533]

1. neither $d_p(x, y)$ nor $d_q(x, y)$ is a metric on X [Option ID = 38133]
2. both $d_p(x, y)$ and $d_q(x, y)$ are metrics on X [Option ID = 38130]
3. only $d_q(x, y)$ is a metric on X [Option ID = 38132]
4. only $d_p(x, y)$ is a metric on X [Option ID = 38131]

Correct Answer :-

- both $d_p(x, y)$ and $d_q(x, y)$ are metrics on X [Option ID = 38130]

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27) Let $f(x) = x \sin x$, $x \in \mathbb{R}$. Then $|f|$ is

[Question ID = 26030]

1. differentiable at $x = \pi$ [Option ID = 44117]
2. differentiable at $x = 0$ [Option ID = 44115]
3. uniformly continuous on $\mathbb{R}$ [Option ID = 44118]
4. differentiable at $x = -\pi$ [Option ID = 44116]

Correct Answer :-

- differentiable at $x = 0$ [Option ID = 44115]

28) Which of the following function f is not uniformly continuous on $\mathbb{R}$

[Question ID = 24541]

1. $f(x) = x + \sin x$ [Option ID = 38163]
2. $f(x) = x + \sin^3 x$ [Option ID = 38165]
3. $f(x) = x^2 + \sin x$ [Option ID = 38164]
4. $f(x) = \sin^2 x$ [Option ID = 38162]

Correct Answer :-

- $f(x) = \sin^2 x$ [Option ID = 38162]

29) Let

$$W = \{(x, y) \in \mathbb{R}^2 \mid xy = 0\},$$
$$X = \{(x, y) \in \mathbb{R}^2 \mid y = 3x\},$$
$$Y = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 = 0\},$$
$$Z = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 0\}.$$

Then the subspaces of $\mathbb{R}^2$ are

[Question ID = 24512]

1. X and Z [Option ID = 38047]
2. Y and Z [Option ID = 38049]
3. W and Y [Option ID = 38046]
4. W and X [Option ID = 38048]

Correct Answer :-

- W and Y [Option ID = 38046]

30) The solution of the Sturm-Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) = 0$, where λ is a constant, is non-trivial for

[Question ID = 24509]

1. all $\lambda > 0$ [Option ID = 38034]
2. all $\lambda < 0$ [Option ID = 38037]
3. $\lambda = 0$ [Option ID = 38036]
4. $\lambda = 1, 4, 9, \ldots$ [Option ID = 38035]

Correct Answer :-

- all $\lambda > 0$ [Option ID = 38034]

31) The maximum and minimum values of the function $f(x, y) = 5x^2 + 2xy + 5y^2$ on the circle $x^2 + y^2 = 1$ denoted by $\max f$ and $\min f$, respectively are

[Question ID = 24539]

1. $\max f = 6,\ \min f = 0$ [Option ID = 38156]
2. $\max f = 6,\ \min f = 4$ [Option ID = 38155]
3. $\max f = \infty,\ \min f = -\infty$ [Option ID = 38157]

4. $\max f = \min f = 5$ [Option ID = 38154]

Correct Answer :-

- $\max f = \min f = 5$ [Option ID = 38154]

32) The general solution of the differential equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ is

[Question ID = 24507]

1. $y = (c_1 + c_2x^2)e^x$ [Option ID = 38027]
2. $y = (c_1 + c_2x)e^{2x}$ [Option ID = 38026]
3. $y = (c_1 + c_2\log x)x$ [Option ID = 38028]
4. $y = (c_1 + c_2\log x)x^2$ [Option ID = 38029]

Correct Answer :-

- $y = (c_1 + c_2x)e^{2x}$ [Option ID = 38026]

33) Let A be a 3×3 matrix over $\mathbb{R}$ with characteristic polynomial $p(\lambda) = \lambda(\lambda - 1)(\lambda - 3)$. Consider the following statements:

(i) The matrix A is not invertible.

(ii) There are three eigen vectors v_1, v_2, v_3 which forms a basis of $\mathbb{R}^3$.

(iii) Each eigen space of A is one dimensional.

(iv) The linear system $(A - 3I)X = B$ has a unique solution for each $B \in \mathbb{R}^3$.

Then

[Question ID = 24511]

1. only (ii) and (iii) are true [Option ID = 38044]
2. only (ii), (iii) and (iv) are true [Option ID = 38043]
3. only (i) and (ii) are true [Option ID = 38045]
4. only (i), (ii) and (iii) are true [Option ID = 38042]

Correct Answer :-

- only (i), (ii) and (iii) are true [Option ID = 38042]

34) The solution of the wave equation

$$u_{tt} - c^2u_{xx} = 0, \quad 0 \le x \le L,\ t > 0$$

with

$$u(0,t) = 0,\ t > 0; \quad u(L,t) = 0,\ t > 0$$

by the method of separation of variables is given by

[Question ID = 24500]

1. $\sum_{n=1}^{\infty} \cos\frac{n\pi x}{L}\left(A_n \sin\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}\right)$ [Option ID = 38000]
2. $\sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \cos\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}\right)$ [Option ID = 37998]
3. $\sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \sin n\pi ct + B_n \cos n\pi ct\right)$ [Option ID = 38001]

4. $\sum_{n=1}^{\infty} \cos \frac{n\pi x}{L} \left(A_n \cos n\pi ct + B_n \sin n\pi ct \right)$ [Option ID = 37999]

Correct Answer :-

- $\sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \left(A_n \cos \frac{n\pi ct}{L} + B_n \sin \frac{n\pi ct}{L} \right)$ [Option ID = 37998]

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35) The values of c_0, c_1 and c_2 so that the formula $\int_{-1}^{1} f(x)dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$ is exact for all polynomials of degree less than or equal to 2 are

[Question ID = 24496]

1. $c_0 = 1,\ c_1 = 1,\ c_2 = 0$ [Option ID = 37983]
2. $c_0 = 1/3,\ c_1 = 4/3,\ c_2 = 1/3$ [Option ID = 37982]
3. $c_0 = 0,\ c_1 = 0,\ c_2 = 1$ [Option ID = 37985]
4. $c_0 = 2/3,\ c_1 = 2/3,\ c_2 = 2/3$ [Option ID = 37984]

Correct Answer :-

- $c_0 = 1/3,\ c_1 = 4/3,\ c_2 = 1/3$ [Option ID = 37982]

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36) Let S, T be linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that $ST = I$, the identity map. Then

[Question ID = 24514]

1. S is one-one but T is not [Option ID = 38055]
2. T is one-one but S is not [Option ID = 38054]
3. Both S and T are one-one [Option ID = 38056]
4. Neither S nor T is one-one [Option ID = 38057]

Correct Answer :-

- T is one-one but S is not [Option ID = 38054]

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37) In cylindrical coordinates (r, θ, z), the Laplace equation $\nabla^2 u = 0$ takes the form

[Question ID = 24501]

1. $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38002]
2. $\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38004]
3. $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38005]
4. $\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38003]

Correct Answer :-

- $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38002]

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38) Let a and b be any two permutations in S_5, the symmetric group on 5 letters. Let $c = a^{-1}(12)a$ and $d = b^{-1}(12)(34)b$. Then

[Question ID = 24518]

1. both c and d are even [Option ID = 38072]

2. both c and d are odd [Option ID = 38073]
3. c is even and d is odd [Option ID = 38071]
4. c is odd and d is even [Option ID = 38070]

Correct Answer :-

- c is odd and d is even [Option ID = 38070]

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39) For a commutative ring R with identity consider the following statements

(i) Let I be an ideal of R such that every element of R not in I is a unit (invertible). Then R/I is a field.

(ii) An ideal I of R is prime if and only if R/I is an integral domain.

(iii) Every non-zero prime ideal of R is maximal.

Then

[Question ID = 24524]

1. only (ii) and (iii) are true [Option ID = 38095]
2. only (i) and (iii) are true [Option ID = 38096]
3. only (i) and (ii) are true [Option ID = 38094]
4. all of (i), (ii) and (iii) are true [Option ID = 38097]

Correct Answer :-

- only (i) and (ii) are true [Option ID = 38094]

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40) Let $\mathcal{A}$ denote the subset $\mathbb{Q} \times \mathbb{Q}$ of $\mathbb{R}^2$ and $\mathcal{U}$ denote the set of all lines in $\mathbb{R}^2$ that intersect with $\mathcal{A}$ in at least two points. Then

[Question ID = 24537]

1. both $\mathcal{A}$ and $\mathcal{U}$ are uncountable [Option ID = 38146]
2. both $\mathcal{A}$ and $\mathcal{U}$ are countable [Option ID = 38147]
3. $\mathcal{A}$ is countable but $\mathcal{U}$ is uncountable [Option ID = 38148]
4. $\mathcal{U}$ is countable but $\mathcal{A}$ is uncountable [Option ID = 38149]

Correct Answer :-

- both $\mathcal{A}$ and $\mathcal{U}$ are uncountable [Option ID = 38146]

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41) A set $X \subseteq \mathbb{R}$ is said to be a null set if for every $\epsilon > 0$ there exists a countable collection $\{(a_k, b_k)\}_{k=1}^{\infty}$ of open intervals such that $X \subseteq \bigcup_{k=1}^{\infty}(a_k, b_k)$ and $\sum_{k=1}^{\infty}(b_k - a_k) \leq \epsilon$. Which of the following set is not a null set?

[Question ID = 24527]

1. Every finite set [Option ID = 38109]
2. $\mathbb{Q}^c$, the set of irrational numbers [Option ID = 38108]
3. $\mathbb{N}$, the set of natural numbers [Option ID = 38106]
4. $\mathbb{Q}$, the set of rational numbers [Option ID = 38107]

Correct Answer :-

- $\mathbb{N}$, the set of natural numbers [Option ID = 38106]

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42) Let f be a bounded Riemann integrable function on $[a, b]$ and F be its indefinite integral. Which of the following is not true?

[Question ID = 24528]

1. F is continuous on $[a, b]$ [Option ID = 38111]
2. F need not be differentiable on $[a, b]$ [Option ID = 38113]
3. F is differentiable on $[a, b]$ and $F'(x) = f(x)$ for every $x \in [a, b]$ [Option ID = 38112]
4. F satisfies Lipschitz's condition [Option ID = 38110]

Correct Answer :-

- F satisfies Lipschitz's condition [Option ID = 38110]

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43) Let $\langle a_n \rangle$ be a bounded sequence of real numbers with $\limsup a_n \neq \liminf a_n$. Consider the following statements

(i) $\lim a_n$ does not exist.

(ii) $\liminf a_n < \limsup a_n$.

(iii) There is a convergent subsequence of $\langle a_n \rangle$.

Then

[Question ID = 24530]

1. all of (i), (ii) and (iii) are true [Option ID = 38121]
2. only (ii) is true [Option ID = 38119]
3. only (i) and (ii) are true [Option ID = 38118]
4. only (ii) and (iii) are ture [Option ID = 38120]

Correct Answer :-

- only (i) and (ii) are true [Option ID = 38118]

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44) The area bounded by the curve and x axis with data

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

using trapezoidal rule is

[Question ID = 24498]

1. 0.0996 [Option ID = 37991]
2. 0.0876 [Option ID = 37990]
3. 0.0745 [Option ID = 37992]
4. 0.0912 [Option ID = 37993]

Correct Answer :-

- 0.0876 [Option ID = 37990]

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45) The series $\sum \frac{(-1)^n}{n^p}$

[Question ID = 24531]

1. converges for all values of p [Option ID = 38124]
2. converges for p > 0, diverges for p ≤ 0 [Option ID = 38122]
3. does not converges for any value of p [Option ID = 38125]
4. converges for p > 1, diverges for p ≤ 1 [Option ID = 38123]

Correct Answer :-

- converges for p > 0, diverges for p ≤ 0 [Option ID = 38122]

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46)

The solution of the differential equations

$$x'(t) = -y + t,$$

$$y'(t) = x - t$$

with c_1 and c_2 as arbitrary constants, is

[Question ID = 26022]

1. $x = c_1 \cos t - c_2 \sin t + t + 1; \quad y = c_1 \sin t + c_2 \cos t - t + 1$ [Option ID = 44086]
2. $x = c_1 \cos t + c_2 \sin t + t + 1; \quad y = c_1 \sin t - c_2 \cos t + t - 1$ [Option ID = 44083]
3. $x = c_1 \cos t - c_2 \sin t + t + 1; \quad y = c_1 \sin t + c_2 \cos t + t + 1$ [Option ID = 44084]
4. $x = c_1 \cos t + c_2 \sin t + t + 1; \quad y = c_1 \sin t - c_2 \cos t + t + 1$ [Option ID = 44085]

Correct Answer :-

- $x = c_1 \cos t + c_2 \sin t + t + 1; \quad y = c_1 \sin t - c_2 \cos t + t - 1$ [Option ID = 44083]

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47) The proof of the fact that the sequence $\left\langle \frac{1}{n} \right\rangle$ converges to zero relies on

[Question ID = 24538]

1. None of these [Option ID = 38153]
2. both completeness and the archimedian properties of $\mathbb{R}$. [Option ID = 38152]
3. only the completeness property of $\mathbb{R}$. [Option ID = 38151]
4. only the archimedian property of $\mathbb{R}$. [Option ID = 38150]

Correct Answer :-

- only the archimedian property of $\mathbb{R}$. [Option ID = 38150]

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48) Which sets are compact?

$$X = \{x^{-1} \mid x \geq 2\} \subseteq \mathbb{R}$$
$$Y = \left\{(x, y) \in \mathbb{R}^2 \mid x^3 + y^3 = 1\right\}$$
$$Z = \left\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{4} + \frac{y^2}{9} = 1\right\}$$

[Question ID = 24532]

1. All of X, Y and Z [Option ID = 38126]
2. Only Y and Z [Option ID = 38127]
3. Only X and Z [Option ID = 38128]
4. Only Z [Option ID = 38129]

Correct Answer :-

- All of X, Y and Z [Option ID = 38126]

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49) Let $\mathbf{F}(x, y, z) = \frac{y}{x^2 + y^2}\mathbf{i} - \frac{x}{x^2 + y^2}\mathbf{j} + \mathbf{k}$ be defined on $D = \{(x, y) \in \mathbb{R}^2 \ : \ x^2 + y^2 > 0\}$. If C denotes the unit circle in xy plane, then

[Question ID = 24540]

1. curl $\mathbf{F} = \mathbf{0}$ in D and $\int_C \mathbf{F}.\mathbf{ds} = \mathbf{0}$ [Option ID = 38161]
2. curl $\mathbf{F} \neq \mathbf{0}$ in D but $\int_C \mathbf{F}.\mathbf{ds} = \mathbf{0}$ [Option ID = 38159]

3. $\text{curl } \mathbf{F} \neq \mathbf{0} \text{ in } D \text{ and } \int_C \mathbf{F}.\mathbf{ds} \neq \mathbf{0}$ [Option ID = 38160]
4. $\text{curl } \mathbf{F} = \mathbf{0} \text{ in } D \text{ but } \int_C \mathbf{F}.\mathbf{ds} \neq \mathbf{0}$ [Option ID = 38158]

Correct Answer :-

- $\text{curl } \mathbf{F} = \mathbf{0} \text{ in } D \text{ but } \int_C \mathbf{F}.\mathbf{ds} \neq \mathbf{0}$ [Option ID = 38158]

50) Let *K* be any subgroup of a group *G* and *H* be the only subgroup of order *m* in *G*. Which of the following is not true?

[Question ID = 24516]

1. *H* is a normal subgroup of *G* [Option ID = 38062]
2. *G = N(H)*, where *N(H)* is the normalizer of *H in G*. [Option ID = 38065]
3. $ab \in H$ implies that $ba \in H$ [Option ID = 38064]
4. *HK* is not a subgroup of *G* [Option ID = 38063]

Correct Answer :-

- *H* is a normal subgroup of *G* [Option ID = 38062]